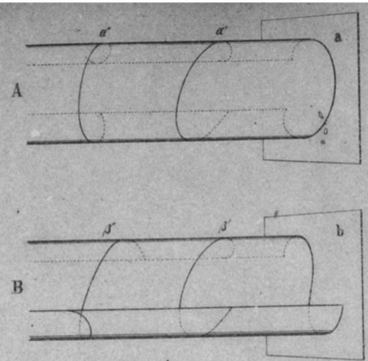

Fig. 1.

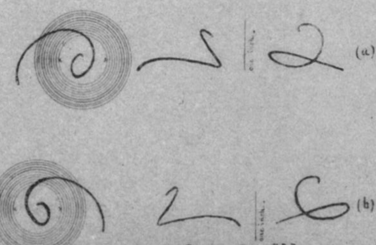

Fig. 2. The Type-Helix,
(a) direct; (b) reversed.

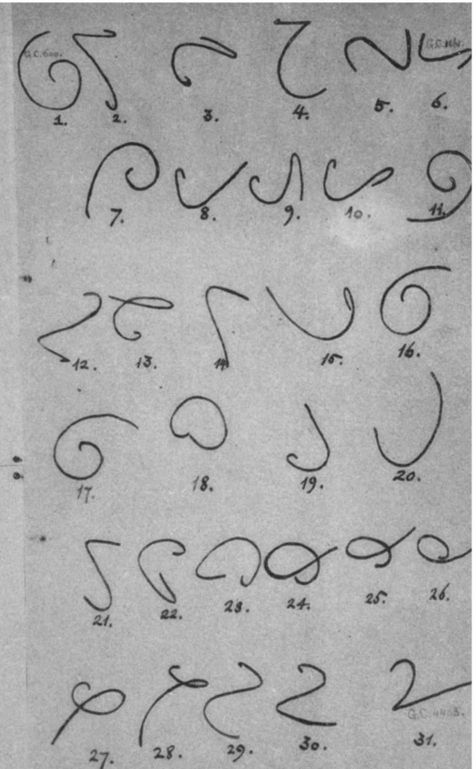

Fig. 3.

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ON THE HELICAL NEBULÆ.

By Edward S. Holden.

The discovery of the helical appearance of the planetary nebula H. iv. 37 (G. C. 4373) at this Observatory in 1888* naturally led to the search for a method which might enable one, in some cases at least, to determine the actual situation of the different branches of a nebula in space of three dimensions from the data afforded by the projection of these branches upon the background of the sky. In general, this problem is hopelessly insoluble by our present means.

I have, however, obtained some interesting results for one class of nebulæ at least, and perhaps the method employed is capable of wider application.

To understand the method, let us consider how it is that we see a nebula (Plate I, Figure 1). The only data that we have are the outlines a of a drawing of the nebula as it is seen projected against the sky. We must conceive the curve a to be the base of a cylinder, A, whose elements are straight lines (rays of light) extending from the projection a to the eye at A. If the curve a is complicated and involved, so will also be the surface of the cylinder A. Any curve whatever which is drawn on the surface of the cylinder (as a', a'',) will be projected into the same curve a on the sky; so that the real nebula in space may be any one of the infinite number of curves which can be drawn on the surface of this particular cylinder; for any such curve will be projected into the curve a. This is true for any and every nebula, as β, b; γ, c; δ, d, etc., etc. The only thing we really know about the form of a nebula, in general, is that it is projected into a certain shape, as a, or b, or c, or d. The problem is to find the true

*See *Monthly Notices* R. A. S. vol. 48, p. 388.

curves, α, β, γ, δ, in space, knowing only the projected curves a, b, c, d.

In order to fix the ideas, let us think of the elongated strings of nebulosity which form the spiral nebulæ.

Before going further, it is necessary to remark that the data (the curves a, b, c, d) are at present to be obtained only from drawings, and hence they are affected by various classes of errors, due to imperfect telescopic, visual and artistic powers. Photographs of nebulæ are subject to a different and less hurtful class of errors, and they are free from personality. When the great telescope is again in a position to photograph the nebulæ, I shall hope to resume this research with better data. For the present I shall take the drawings of Lord ROSSE, of LASSELL, and others, as the best available, and shall not concern myself with any errors remaining in them, but shall treat them as correct, since they are the best we have.

To resume consideration of the special problem in hand, let us again examine Figure 1. The only thing we know about the nebula in space is that its projection on the sky is a. Any curve on the cylinder A *may be* the true shape of the nebula itself. It is the same for another nebula, b, whose curve b is usually different from that of a. *Any* curve on the surface of B will be projected into b. In general, the shapes of the two cylinders are so utterly different that no two identical curves, α', β', can be drawn on their surfaces.

Now, if we should find a pair of curves, a, b, whose cylinders, A, B, are of such a shape that the same curve *can* be drawn on their surfaces, then there is a certain probability that this identical curve is, in fact, the true shape of each nebula in space. If, again, we can find another nebula, c, whose cylinder, C, is so similar to that of a that like curves can be drawn on the three surfaces, A, B, C, then there is a still greater probability that the identical curve on the three surfaces, A, B, C, is, in fact, the true shape of these three nebulæ, a, b, c, in space. If we find another nebula, d, whose cylinder, D, is of such a shape that we can also draw the same curve on *its* surface, there is a much higher probability that this one curve really represents the true shape of all four nebulæ, a, b, c, d, in space.

As we get more and more examples, all fulfilling the same condition, the probability that we have obtained the true shape of the nebulous form in space is very rapidly increased; and

by finding enough examples we may increase the probability to essential certainty; and still more so, if one curve, and only one, can be found which is common to all the projecting cylinders.

We may attack this problem practically, by seeking through trials for a single curve, Φ, which by projection at various angles and in various positions will give all the differing curves, a, b, c, d, e, ... z. If such a curve can be found (by trial), and if only one such curve can be found, it will become more and more probable that Φ is, in fact, the true curve of each nebula, α, β, γ, δ, ... ω, in proportion as more and more curves, a, b, c, d, ... accurately correspond to the different projections of this type curve, Φ. The idea of such types has been suggested to me by observations of nebulæ with the great telescope, and I have partially discussed it in *Himmel und Erde*, for June, 1889, page 503 *et seq.*

I proceed to give what seems to be the type curve of a certain family of spiral nebulæ. The accompanying Figure 2 shows several representations of a helix of wire, which I have found by trial to be capable of being projected into the shape of each one of the following nebulæ. Figure 2 also gives a scale photographed at the same time as the wire model. The diameter of the smallest circle of the scale is one inch, and the circles are successively $\frac{1}{10}$ of an inch greater in diameter. One inch is also marked near each of the vertical projections.

I give in Figure 3 a selection from projections of the type-helix of Figure 2, which were made by placing the wire model in a beam of parallel rays and tracing its shadow on a plane. Most of the comparisons of drawings of nebulæ with the type-helix have been made by placing the eye vertically over the plane of the paper and by moving the wire helix (its origin nearly always touching the paper in the nucleus of the nebula) until the projection of the helix accurately covered the drawing of the nebula. Usually the model must be applied n different times for a nebula with n branches. I have found no case in which this helix will fit one branch of a nebula without fitting every other branch also.

I give in what follows a few comparisons of this type-helix with drawings of nebulæ, and I begin with the admirable series of drawings given by Mr. Lassell in *Mem.* R. A. S., vol. 36 :

LASSELL'S FIGURE.	G. C. No.	REMARKS.
2	600	The outlines of this nebula have been exactly reproduced (in our Fig. 3, No. 1). [The axis of the type-helix is in position angle 280°, and altitude above paper 70° to 75°.]
3	604	Ditto (when Lassell's figure is reversed).
9	1511	Ditto. (Compare our Fig. 3, No. 3.)
12 (a)	1861	Ditto (in our Fig. 3, No. 6); (b) compare the last drawing of Fig. 2. The nucleus of the nebula is probably due to a crossing of two loops of the helix.
12 (b)	1861	The outlines can be reproduced. (Compare our Fig. 3, Nos. 13, 24, 25.)
15	2373	The loop and the following edge of LASSELL's drawing can be exactly reproduced. (Compare our Fig. 3, Nos. 15, 19.)
16	2838	The axis of the main curve of the drawing has been exactly reproduced. (Compare our Fig. 3, No. 20.)
17	2890	Both these figures have been accurately reproduced. Each branch is a projection of the type-helix. (Compare our Fig. 3, Nos. 11, 16, 17.) Inner spiral, position angle 120°, altitude of axis 80° to 85°; outer spiral, position angle 120°, altitude 80°. If we match the inner spiral and then revolve the type-helix, keeping its axis in the same plane, about 90° in the direction S W N E the outer spiral will be matched.
27	3572 M 15	All the principal branches have been accurately reproduced; one application of the type-helix for each branch. (Compare our Fig. 3, Nos. 1, 7, 11, 16, 17.) Inner spiral, P = 150°, Alt. = 85° to 90°; outer spiral, P = 150°, Alt. = about 80°. Revolve type-helix nearly 180° from the position where it matches the outer spiral in the direction N W S E, and it will match the inner spiral.
28	3606	When this drawing is reversed the three branches can be exactly reproduced by three applications of the type-helix. (Compare our Fig. 3, Nos. 8, 14, 15, 19, 20.) Is the nucleus due to the crossing of two branches of the helix?
29	3614	When this is reversed its two branches can be reproduced by two applications of the type. (Compare our Fig. 3, Nos. 5, 6, etc.)
33	4403	(The Omega nebula.) The axes of the loop and of the straight following part can be exactly reproduced. (Compare our Fig. 3, No. 31.)

N. B.—Note that the position angle of the axis of the type-helix is the same for both spirals of G. C. 2890; and for both spirals of G. C. 3572.

COMPARISONS WITH LORD ROSSE'S DRAWINGS IN THE PHILOSOPHICAL TRANSACTIONS, 1861.

FIG.	G. C.	REMARKS.
9	888	*h.* 327. (Compare our No. 1. etc.)
10	532	*h.* 131. This can be accurately reproduced when it is reversed and its scale changed suitably.
13	2053	*h.* 689. Ditto.
15	2216–17	*h.* 765–6. (Compare our Nos. 2, 3, 4, 12, 13, 28, 31.)
16	2377	*h.* 857. (Compare our Nos. 1, 7, 11, 16, 17, etc.)
18	2670–1	*h.* 1052–3. (Compare our Nos. 2, 3, 12, etc.)
19	2680	*h.* 1061. Can be reproduced.
21	2870	*h.* 1196. (Compare our Nos. 3, 9 (reversed), 12, 29, 30, etc.)
23	3341–2	*h.* 1306–8. (Compare our Nos. 5, 6, 21, etc., and 11, etc.)
24	3085	*h.* 1337. (Compare Nos. 2, 14, twice applied.)
25	3151	*h.* 1385. (Compare our Nos. 23, 24; and notice the opening on the lower side of the figure (as in Fig. 23) and the brightening of the nebula just above this (as in Fig. 23) where the right-hand hook bends back.)
26	3189–90	*h.* 1414–15. (Compare our Nos. 5, 6, 21, 29, 30.)
28	3511	*h.* 1589. (Compare our Nos. 5, 6, etc.)
29	3615	*h.* 1650. (Compare our No. 11, reversed.)
32	4160	*h.* 1946. (Compare our No. 1, etc.)
36	4594	*h.* 2084. If this drawing be reversed, each of the four branches can be accurately represented by projections of the type-helix. I have made a wire model of this nebula.*
41	4971	*h.* 2245. (Compare our No. 1, etc., reversed.)

* Before the present investigation was begun I succeeded in making a model of this nebula of four branches, starting on the assumption that each of the four branches was produced by the projection (at four different angles) of one and the same curve in space. I finally succeeded in bending a wire so that when it was held in four different positions (the origin of the helix always touching the nucleus), the four projections accurately covered the four branches as they are laid down in the drawing. I then laid this model to one side and constructed a type curve from the nebulæ G. C. 600, the great Nebula G. C. 3572 (M. 51) and others. This second type curve was then applied (reversed) to the nebula 4594, and it was found to accurately represent it, and to be the same curve as the one first constructed. Probably in this case, as in others, the conviction that the real type of the nebula has been discovered is more strongly brought home to the person who has actually constructed the models and found them to exactly represent the pictures, than to one who merely reads an account of how the experiment was conducted. The only ambiguity in my model of this nebula is due to the fact that it is impossible to decide on which side of the plane of projection any or all of the branches are situated. We know the real shape of each branch, but we do not know whether it lies on the hither or on the farther side of the plane of projection.

COMPARISON WITH LORD ROSSE'S DRAWINGS IN THE SCIENTIFIC TRANSACTIONS ROYAL DUBLIN SOCIETY, VOL. II.

PLATES.	G. C.	REMARKS.
I.	1202	(Compare our Figs. 24, 25) ??
I.	1267	(Compare our Figs. 14, 15, 19, etc., reversed.)
I.	1519	This can be accurately reproduced.
II.	1520	Ditto.
III.	1861–3	The principal curves in these nebulæ, ditto.
IV.	3572	Ditto.
V.	4561	(Compare our Figs. 9, 10, the middle parts only.)
VI.	4403	The axes of this can be accurately reproduced. (See our Fig. 31.)

It is unnecessary to give more examples. Indeed, the cases already given include nearly all the spiral nebulæ. Those just referred to are sufficient to exhibit the whole evidence to any one who will construct for himself a type curve from the data in Figure 2, and who will go over the comparisons with the plates as above outlined. The spirals of *Nebula Orionis* are probably of the type just given, also. The case of the *Omega* nebula (G. C. 4403) is very striking. I have also found remarkable analogies in various spiral streams of stars.

It may be objected to the suggestions given above that the forms of the nebulæ are so indefinite that a very great latitude is allowed in matching the drawings with the projections of any particular type curve. This is undoubtedly true. The only remedy for it is to obtain better representations of the nebulæ themselves by photographic means.

A second objection is that Figure 3 shows that a particular spiral, once assumed, may be projected into many forms, and that these might be sufficiently varied to be fitted to a comparatively small number of objects out of the many thousands of known nebulæ. To this it may be said that it is undoubtedly true that the projection of many different curves can be made to fit a certain number of the drawings referred to. Still, it appears to me, after trials, that the helix of Figure 2 comes nearer to being the type curve of the nebulæ in question than any other that I can now construct. It certainly will need to be corrected, but it seems to be a good first approximation.

The difficulty of improving it can be best appreciated by making the trial.

Again, it must be remembered that while there are many thousands of nebulæ, there are only comparatively few spiral nebulæ, and that the type curve fits a very great percentage of these, while it cannot be tortured into a resemblance to other nebulæ not spiral.

If the helix given in Figure 2 is indeed the type of a certain class of nebulæ, many interesting questions may receive a solution. For example, what are the directions in space of the *axes* of these different nebulæ? Is there anything systematic in these directions? What is the law of the force by which particles of matter are expelled from (or attracted to?) the central nucleus? Have we here in the nebulæ different types of spirals somewhat analogous to the different types of comets' tails so ably discussed by Professor BREDICHIN?

Some of the parts of these nebulæ must be approaching the earth, some receding from it. Can we by the spectroscope discriminate between such motions?

A suggestion which holds out even the hope of successfully attacking such problems is not without its value, and I have, therefore, no hesitation in presenting the foregoing paper in its present incomplete form.

EDWARD S. HOLDEN.

LICK OBSERVATORY, July 12, 1889.

ON THE ORBIT OF COMET BARNARD (1889, JUNE 23).

BY A. O. LEUSCHNER.

From Mr. BARNARD'S observations of June 23, 24, 25, I have deduced the following elements:

$$T = 1889, \text{ June } 20.1480 \text{ G. m. t.}$$

$$\left.\begin{aligned} \Omega &= 271^\circ\ 4'.1 \\ \omega &= 59^\circ\ 20'.7 \\ i &= 31^\circ\ 14'.6 \end{aligned}\right\} 1889.0$$

$$\log q = 0.04236$$

Obsd.—Computed; $\Delta\lambda \cos\beta = -0'.3$, $\Delta\beta = 0'.0$.